TABLE 2. Influence of Form of Moisture Bonding and Drying Temperature on the Kinetics of the Process of Thread Deformation

Experimental temperature, °K	Moisture content corresponding to beginning of deformation, %	Moisture content corresponding to maximum rate of deformation, %	Maximum rate of deformation %/min	Relative deformation of threads, %			
				maximum	corresponding to elimination of moisture from macropores	corresponding to elimination of moisture from micropores	corresponding to elimination of adsorbed moisture
313	140	20-45	0.25	5.9	3.2	1.4	1.3
353	165	30-60	1.03	6.5	4.0	1.8	0.7
393	170	30-60	1.60	6.9	4.3	1.9	0.7

Thus, shrinkage effects in viscose threads when dried are governed principally by the forms of bonding of the water eliminated, and the stabilization of the porous structure of the threads is governed by the number of repeated drying and wetting operations. Cyclical wetting–drying thus generates a significant reduction in the volumes of the macropores and micropores.

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INTERRELATED HEAT AND MASS TRANSFER IN A FLUIDIZED BED IN AN OSCILLATING MODE

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The problem of the interrelated heat and mass transfer in a fluidized bed in an oscillating mode is formulated with allowance for the circulating motion of the particles, and its solution is obtained with some assumptions.

As is known, a specific property of heat exchange in a fluidized bed consists in the fact that the particles of the bed undergo a brief temperature pulse in a thin layer near the grid owing to heat transfer from the fluidizing agent to the particle surface. The temperature of a particle falls with greater distance from the gas-distributing grid because of the effective heat conduction of the bed and the conductive propagation of heat into the particle, and starting with a certain height the bed becomes isothermal. Thus, the temperature of a particle in the layer near the grid differs from its temperature in the remaining volume. This temperature difference can reach considerable amounts. For example, according to the experimental data of [1] the surface of a moist grain particle is heated by 20°C in 0.2 sec, by 30° in 0.3 sec, and by 40° after 0.5 sec. Upon further heating the temperature difference between the surface and center decreases, although even after 3 sec it was still

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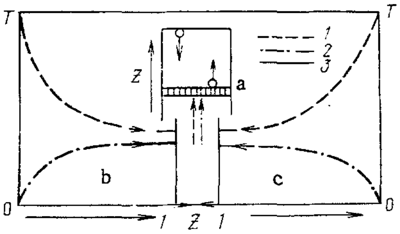

Fig. 1. For the formulation of the boundary conditions: a) diagram of mutual motion of particles and fluidizing medium; b) direct flow; c) counterflow; 1) hot air; 2) cold air; 3) direction of particle motion.

10° despite the increase in temperature at the center of the particle (the temperature of the heat-transfer agent $t_h = 280°C$). According to [2], for a moist grain $\lambda = 0.116$ W/m·deg, while $a = 0.7 \cdot 10^{-3}$ m²/h; we take $\alpha = 175$ W/m²·deg, and then for d = 4 mm and $\tau = 1$ sec we have Bi = 3.0 and Fo = 0.05. According to [3] the excess temperature at the surface of the particle is 48.5° ($\vartheta_0 = 20°C$), while with Bi = 4 and Fo = 0.005 ($\tau = 0.1$ sec) it is 29°C (the temperature of the heat-transfer agent $t_h = 130°C$); in both cases the temperature at the center of the particle is practically constant. These data pertain to heating of the particle, since because Lu ≪ 1, it is natural to assume that the evaporation of moisture or its redistribution over the cross section of the particle does not occur during a brief heat pulse.

At the same time, in calculations it is valid to assume that the final temperature of the material is approximately equal to the temperature of the heat-transfer agent leaving the bed [4]. For heat-sensitive products, however, orientation to the final temperature of the heat-transfer agent without allowance for possible overheating of the particles near the grid can lead to irreversible changes degrading the properties of the material of the particles (the death of living organisms, decomposition of the material, the release of undesirable substances, melting or fusion, cracking of the particles because of thermal stresses, etc.). This pertains especially to those particles for which one cannot neglect the temperature gradient over the cross section, since during a brief pulse the surface layers are subjected to overheating, while the integral-average temperature over the volume of the particle can remain low and its final value may not exceed the permissible limit. Moreover, one must consider that the particle is repeatedly subjected to such overheating. An examination of the dynamics of the heating of a particle with respect to the height is especially essential for an oscillating mode of drying, since with this method of heat supply one of the elements of the intensification of the process consists in raising the initial temperature of the heat-transfer agent during the heating periods.

Formulation of the Problem

The basic equations of interrelated heat and mass transfer [5] with boundary conditions of the third kind are also valid for a fluidized bed. In the majority of cases of practical importance, however, the complete system of equations and some boundary conditions can be simplified. Because of the smallness of the heat-gradient coefficient δ for the majority of materials [6] and the considerable time lag of moisture field relative to the temperature field for heat-sensitive materials [7], the values of the Posnov and Lykov numbers are very small, as an analysis shows [8]. Because of this one can also assume that the moisture content of a particle varies only in time, remaining constant over the height of the bed. In the drying of heat-sensitive materials it is important to achieve conditions such that deepening of the evaporation zone is prevented, i.e., so that the moisture within the particle moves in the form of a liquid. In this case, as is known, the phase-conversion criterion is $\xi = 0$.

In the thermal treatment of materials in a fluidized bed the system of equations must be closed by the equation of heat balance for the fluidizing medium, which has a different form for continuous and periodic processes. The boundary conditions at the particle surface are also changed in this case.

A periodic drying process is characterized by continuous variation with time in the temperature and moisture content of the entire collection of particles of the bed and in the coefficients of heat and mass transfer. In this case the temperature of the fluidizing medium is a function of the coordinates and time.

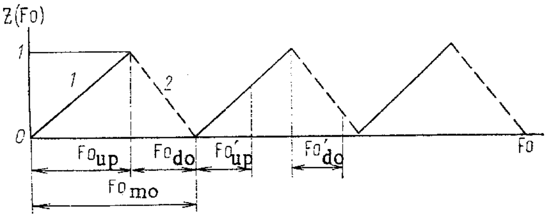

Fig. 2. Graph of variation in the function Z(Fo) [Eq. (5)]: 1) $Z_{up} = \bar{v}_Z^+ \cdot Fo'_{up}$; 2) $Z_{do} = 1 - \bar{v}_X^- \times Fo'_{do}$.

A continuous drying process in a fluidized bed is characterized by constancy of the statistical-average moisture of the entire bed in a given cross section with time, and the temperature of the heat-transfer agent is also constant in time in a constant cross section of the bed. However, for each separate element of the bed the process is unsteady, since the brief temperature pulse in the layer near the grid leads to the formation of a transient temperature field of the moist particle, and therefore a system of equations of interrelated heat and mass transfer is valid for the particles of the bed. In this case $dt/d\tau = 0$ and the balance equation for the fluidizing medium can be written as [9, 10]

$$\alpha F[t(h, \tau) - \vartheta(R, h, \tau)]\,dh = -Lc_p dt. \tag{1}$$

Integrating Eq. (1) from $t(0, \tau)$ to $t(h, \tau)$ and from 0 to h, in dimensionless form we obtain

$$T(Z, \mathrm{Fo}) = \Theta(1, 1, \mathrm{Fo}) + [T(0, \mathrm{Fo}) - \Theta(1, 1, \mathrm{Fo})]\exp(-pZ). \tag{2}$$

Although Eq. (2) is obtained with the condition of constancy of the particle temperature over the height of the bed, it is confirmed well by experimental data. Here the dimensionless flux of moisture can be represented as follows:

$$\mathrm{Ki}_m(\mathrm{Fo}) = \mathrm{Bi}_m[1 - U(1, \mathrm{Fo})]. \tag{3}$$

As is known, the motion of particles in a fluidized bed has a chaotic random nature. In this case the velocity of particle motion can be represented as a superposition of determined and random components.

To allow for the effect of the particle motion in the bed on the variation in particle temperature over the height of the bed we make the following assumptions:

1) the hydrodynamic environment is identical in all vertical cross sections of the bed; with this condition the random velocity component of particle motion is reflected only in the average value of the determined component, since the point of a given horizontal cross section of the bed at which the particle is found makes no difference at all;
2) the principle of equivalence of the choice of an arbitrary particle of the bed occurs; consequently, the result of an analysis of the heat and mass transfer for any particle of the bed can be extended to any other particle of the bed.

These assumptions make it possible to formulate the following phenomenological model of heat exchange in a fluidized bed with allowance for the circulating motion of the particles; one considers a fixed particle for which the heat exchange takes place with a medium having a temperature which varies in accordance with Eq. (2). Consequently, Eq. (2) reflects the presence of an ensemble of particles. Despite the chaotic nature of particle motion in the bed, the determined component has a clearly expressed vertical reciprocating nature (the circulating motion of the particles [11-13]). Since the gas motion occurs in only one direction (upward), the heat exchange between the fluidizing agent and the particles proceeds by the direct-flow principle during the upward motion of the particles (from the gas-distributing grid to the top section of the bed) and by the counterflow principle during the downward motion of a particle (from the top section of the bed to the gas-distributing grid), as illustrated in Fig. 1.

Thus, Eq. (2) must be written as follows:

$$T(Z, \mathrm{Fo}) = \begin{cases} \Theta(1, 1, \mathrm{Fo}) + [T(0, \mathrm{Fo}) - \Theta(1, 1, \mathrm{Fo})]\exp(-pZ), & 0 \leqslant Z \leqslant 1, \\ \Theta(1, 1, \mathrm{Fo}) + [T(0, \mathrm{Fo}) - \Theta(1, 1, \mathrm{Fo})]\exp(-pZ), & 1 \leqslant Z \leqslant 0. \end{cases} \tag{4}$$

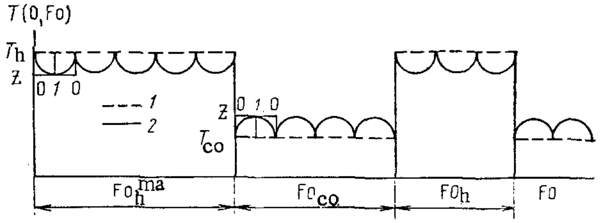

Fig. 3. Variation in temperature of fluidizing medium: 1) initial; 2) during filtration through the bed.

Consequently, the vertical coordinate Z is a function of time, i.e., Z(Fo). Since reciprocating particle motion occurs, the function Z(Fo) is periodic. In the general case, as is shown in [14, 15], for example, the average particle velocities upward $\bar{v}_a^+$ and downward $\bar{v}_{\bar{a}}$ are different. Then the perilodic function Z(Fo), as follows from Fig. 2, can be represented in the form

$$Z(\mathrm{Fo})=\begin{cases}\dfrac{\mathrm{Fo}}{\mathrm{Fo}_{up}}-m\,\mathrm{Fo}_{mo}, & m\,\mathrm{Fo}_{mo}<\mathrm{Fo}<m\,\mathrm{Fo}_{mo}+\mathrm{Fo}_{up}\\ -\dfrac{\mathrm{Fo}}{\mathrm{Fo}_{do}}+(m+1)\,\dfrac{\mathrm{Fo}_{mo}}{\mathrm{Fo}_{do}}, & m\,\mathrm{Fo}_{mo}+\mathrm{Fo}_{up}<\mathrm{Fo}<(m+1)\,\mathrm{Fo}_{mo}\end{cases} \tag{5}$$

$$(m=0,\ 1,\ 2,\ 3\ldots).$$

$$Z(\mathrm{Fo})=0 \text{ for } \mathrm{Fo}<0 \text{ and } \mathrm{Fo}>(m+1)\,\mathrm{Fo}_{mo};\ \mathrm{Fo}_{mo}=\mathrm{Fo}_{up}+\mathrm{Fo}_{do}$$

For convenience in the later calculations we introduce the intermediate value of the quantities: $0 \leq \mathrm{Fo}'_{up} \leq \mathrm{Fo}_{up}$; $0 \leq \mathrm{Fo}'_{do} \leq \mathrm{Fo}_{do}$. With known average values of the dimensionless velocities of particle motion upward $\bar{v}_z^+$ and downward $\bar{v}_{\bar{z}}$ we have the following: $\mathrm{Fo}_{up} = Z/\bar{v}_z^+$; $\mathrm{Fo}_{Do} = (1-Z)/\bar{v}_{\bar{z}}$ If one takes into account Fo'_{up} and Fo'_{do}, then the limits of Fo take the following values for upward particle motion (ascent) and downward particle motion (descent). For the last period of particle ascent the top line in (5) is written as follows: $m\mathrm{Fo}_{mo} < \mathrm{Fo} < m\mathrm{Fo}_{mo} + \mathrm{Fo}'_{up}$; for particle descent in the last period the bottom line in (5) has the following form: $m\mathrm{Fo}_{mo} + \mathrm{Fo}_{up} < \mathrm{Fo} < m\mathrm{Fo}_{mo} + \mathrm{Fo}_{up} + \mathrm{Fo}'_{do}$. Thus, the position of a particle along the height of the bed is determined by the value of Fo'_{up} and Fo'_{do} during its ascent and descent, respectively. In a real fluidization process the particle does not always reach its extreme positions. In calculations for the upward motion of a particle one can consequently be limited to some value $\mathrm{Fo}'_{up} < \mathrm{Fo}_{up}$, especially since the bed becomes isothermal after the section of thermal stabilization. As for the downward motion of a particle, the particle temperature for $\mathrm{Fo}'_{do} \approx 0$ is of the greatest interest. With allowance for (5), Eq. (4) can be written as follows:

$$T(Z,\mathrm{Fo})=\Theta(1,1,\mathrm{Fo})+[T(0,\mathrm{Fo})-\Theta(1,1,\mathrm{Fo})]\exp[-pZ(\mathrm{Fo})]. \tag{6}$$

In an oscillating mode of drying the initial temperature of the heat-transfer agent is a periodic function of time [6]:

$$T(0,\mathrm{Fo})=\begin{cases}T_h, & 0<\mathrm{Fo}<\mathrm{Fo}_h^{ma},\\ T_{co}, & \mathrm{Fo}_h^{ma}+k\,\mathrm{Fo}_c<\mathrm{Fo}<\mathrm{Fo}_h^{ma}+k\,\mathrm{Fo}_c+\mathrm{Fo}_{co},\\ T_h, & \mathrm{Fo}_h^{ma}+k\,\mathrm{Fo}_c+\mathrm{Fo}_{co}<\mathrm{Fo}<\mathrm{Fo}_h^{ma}+(k+1)\,\mathrm{Fo}_c,\ (k=0,\ 1,\ 2,\ 3\ldots),\end{cases} \tag{7}$$

$$T(0,\ \mathrm{Fo})=0 \text{ for } \mathrm{Fo}<0 \text{ and } \mathrm{Fo}>(k+1)\,\mathrm{Fo}_c.$$

The preliminary heating of the material is taken into account in Eq. (7).

In each period of heating or cooling a particle repeatedly undergoes upward and downward motion, but a fully determined number of times. Therefore, the quantity m in Eq. (5), which characterizes the number of ascents and descents of a particle in the bed in each of the periods of heating and cooling, is a finite quantity. We introduce the intermediate quantities $0 \leq \mathrm{Fo}'_{co} \leq \mathrm{Fo}_{co}$ and $0 \leq \mathrm{Fo}'_h \leq \mathrm{Fo}_h$. A simple relationship exists between Fo'_{co}, Fo'_{up}, and Fo'_{do}: $\mathrm{Fo}'_{co} = m\mathrm{Fo}_{mo} + \mathrm{Fo}'_{up}$ for periods of particle ascent and $\mathrm{Fo}'_{co} = m\mathrm{Fo}_{mo} + \mathrm{Fo}_{up} + \mathrm{Fo}'_{do}$ for periods of descent. The same relations are also valid for Fo'_h. With allowance for the intermediate values the last two lines in (7) take the following values for the last cycles of cooling and heating: $\mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c < \mathrm{Fo} < \mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c + \mathrm{Fo}'_{co}$ ($k = 0, 1, 2, 3, \ldots$) and $\mathrm{Fo}_h^{ma} + (k-1)\mathrm{Fo}_c + \mathrm{Fo}_{co} < \mathrm{Fo} < \mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c$ ($k = 1, 2, 3\ldots$) for the last cooling cycle (Fo_{co} instead of Fo'_{co} for all but the last cycle); $\mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c < \mathrm{Fo} < \mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c + \mathrm{Fo}_{co}$ and $\mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c + \mathrm{Fo}_{co} < \mathrm{Fo} < \mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c + \mathrm{Fo}_{co} + \mathrm{Fo}'_h$ ($k = 0, 1, 2, 3\ldots$) for the last heating cycle (Fo_h instead of Fo'_h for all but the last cycle).

Thus, with allowance for the conditions formulated above, we arrive at the following initial system of equations:

$$\frac{\partial\Theta(X, Z, \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2\Theta(X, Z, \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X}\cdot\frac{\partial\Theta(X, Z, \mathrm{Fo})}{\partial X}, \tag{8}$$

$$\frac{\partial U(X, \mathrm{Fo})}{\partial \mathrm{Fo}} = \mathrm{L}\left[\frac{\partial^2 U(X, \mathrm{Fo})}{\partial X^2} + \frac{\Gamma}{X}\cdot\frac{\partial U(X, \mathrm{Fo})}{\partial X}\right], \tag{9}$$

$$U(X, 0) = 0, \quad \Theta(X, Z, 0) = 0, \tag{10}$$

$$-\left.\frac{\partial U(X, \mathrm{Fo})}{\partial X}\right|_{X=1} + \mathrm{Bi}_m[1 - U(1, \mathrm{Fo})] = 0, \tag{11}$$

$$\left.\frac{\partial\Theta(X, Z, \mathrm{Fo})}{\partial X}\right|_{X=1} + \mathrm{Bi}_m[1 - U(1, \mathrm{Fo})]\,\mathrm{Ko}\,\mathrm{Ly} - \mathrm{Bi}\{\Theta(1, 1, \mathrm{Fo})$$
$$-\Theta(1, Z, \mathrm{Fo}) + [T(0, \mathrm{Fo}) - \Theta(1, 1, \mathrm{Fo})]\exp[-pZ(\mathrm{Fo})]\} = 0, \tag{12}$$

$$\left.\frac{\partial U(X, \mathrm{Fo})}{\partial X}\right|_{X=0} = 0, \quad \left.\frac{\partial\Theta(X, Z, \mathrm{Fo})}{\partial X}\right|_{X=0} = 0. \tag{13}$$

Analytical Solution

Since the continuous process is of the greatest interest, we will be confined to a solution for only this case. First we solve an auxiliary problem – Eq. (9) with the conditions (10), (11), and (13) for U. Applying a Laplace transform, with allowance for (10) we obtain the following in the transform region in place of (9):

$$s\bar{U}(X, s) = \mathrm{Ly}\left[\frac{d^2\bar{U}(X, s)}{dX^2} + \frac{\Gamma}{X}\cdot\frac{d\bar{U}(X, s)}{dX}\right]. \tag{14}$$

Let us consider the equation

$$\frac{d^2F_\Gamma(X)}{dX} + \frac{\Gamma}{X}\cdot\frac{dF_\Gamma(X)}{dX} = F_\Gamma(X) \tag{15}$$

with the condition

$$\left.\frac{dF_\Gamma(X)}{dX}\right|_{X=0} = 0. \tag{16}$$

We represent $F_\Gamma(X)$ in the form

$$F_\Gamma(X) = \sum_{k=0}^{\infty} a_k X^k. \tag{17}$$

Substituting (17) into (15) with allowance for (16), we obtain the following expression for $F_\Gamma(X)$:

$$F_\Gamma(X) = \sum_{k=0}^{\infty}\frac{X^{2k}}{(2k)!!\,(2k - 1 + \Gamma)!!}. \tag{18}$$

We note that a solution of an equation of the type of (8) was obtained in another form in [17, 18]. The function $F_\Gamma(X)$ is even. Particular cases of the function $F_\Gamma(X)$ are

$$F_0(X) = \mathrm{ch}(X);\ F_1(X) = I_0(X);\ F_2(X) = \frac{\mathrm{sh}\,X}{X};\ F_\Gamma(0) = 1.$$

With allowance for (13), the solution of Eq. (14) in accordance with (15) can be written in the form

$$\bar{U}(X, s) = A^*(s)\,F_\Gamma\left(\sqrt{\frac{s}{\mathrm{Ly}}}X\right), \tag{19}$$

where the integration constant A*(s) is determined from the condition (11).

Let us now turn to the solution of the main problem. In the transform region we have in place of (8)

$$s\bar{\Theta}(X, Z, s) = \frac{d^2\bar{\Theta}(X, Z, s)}{dX^2} + \frac{\Gamma}{X}\cdot\frac{d\bar{\Theta}(X, Z, s)}{dX}. \tag{20}$$

The solution of Eq. (20) is written analogously to Eq. (19) in the form

$$\bar{\Theta}[X,\ Z(s),\ s] = A[Z(s),\ s]\,F_{\Gamma}(\sqrt{s}X). \tag{21}$$

In the drying of heat-sensitive materials, as is known, the maximum final temperature must not exceed the limiting permissible temperature. With preliminary heating [19] in the top section of the bed ($Z = 1$) the temperature $\Theta(1,\ 1,\ \mathrm{Fo})$ of the material is approximately equal to the maximum permissible temperature Θ^* and with a certain approximation it can be taken as a constant quantity. Consequently,

$$\Theta(1,\ 1,\ \mathrm{Fo}) = \Theta^* = \mathrm{const.} \tag{22}$$

The solution of the problem will thus reflect the law of variation of the temperature of the material from the value received in the temperature pulse at the grid to the maximum permissible value Θ^*. Consequently, the calculation comes down to the determination of $\Theta(X,\ Z,\ \mathrm{Fo})$ for $0 \le Z \le 1$ with a known $\Theta(1,\ 1,\ \mathrm{Fo})$.

Here we neglect the small temperature changes during the heating and cooling periods for $\Theta(1,\ 1,\ \mathrm{Fo})$. With allowance for (22), the condition (12) is rewritten as follows:

$$\left.\frac{\partial\Theta}{\partial X}\right|_{X=1} + \mathrm{Bi}_m[1 - U(1,\ \mathrm{Fo})]\,\mathrm{Ko\,Ly} - \mathrm{Bi}\,\Theta^*[1 - \exp(-pZ(\mathrm{Fo}))]$$
$$+ \mathrm{Bi}\,\Theta(1,\ Z,\ \mathrm{Fo}) - \mathrm{Bi}\,T(0,\ \mathrm{Fo})\exp[-pZ(\mathrm{Fo})] = 0. \tag{23}$$

Performing the intermediate calculations and converting to the inverse transform on the basis of the expansion theorem, we obtain the final solution in the following form:

$$\Theta(X,\ \mathrm{Fo}) = \Theta^* - \sum_{n=1}^{\infty} \frac{\mathrm{Ko}^* F_{\Gamma}(i\nu_n X)\,F_{\Gamma}(i\overset{*}{\nu}_n X)}{\psi_1'(\nu_n)}\exp(-\nu_n\,\mathrm{Fo}) + \left[\frac{\mathrm{Ko}^* F_{\Gamma}(i\overset{*}{\mu}_n X)\,F_{\Gamma}(i\mu_n X)}{\psi_1'(\mu_n)} - \left(\frac{\mathrm{Bi}^* F_{\Gamma}(i\mu_n X)}{\mu_n^2}\right.\right.$$
$$\left.\left. - \frac{\Omega_1 F_{\Gamma}(i\mu_n X)\,\mathrm{Fo}_{\mathrm{up}}}{p - \mu_n^2 \mathrm{Fo}_{\mathrm{up}}} + \frac{\Omega_2 F_{\Gamma}(i\mu_n X)\,\mathrm{Fo}_{\mathrm{do}}}{p + \mu_n^2 \mathrm{Fo}_{\mathrm{do}}}\right)\frac{1}{\psi_2'(\mu_n)}\right]\exp(-\mu_n^2 \mathrm{Fo}), \tag{24}$$

where $\mathrm{Ko}^* = \mathrm{Bi}_m^2\mathrm{KoLy}$; $\mathrm{Bi}^* = \mathrm{Bi}_m\mathrm{KoLy} - \mathrm{Bi}\,\Theta^*$. In Eq. (24), Θ is written as a function of the two variables X and Fo, since Z is a function of Fo. In (24), the Fourier number takes on different values: $\mathrm{Fo} = \mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c + \mathrm{Fo}_{co}'$ for a cooling period and $\mathrm{Fo} = \mathrm{Fo}_h^{ma} + k\mathrm{Fo}_c + \mathrm{Fo}_{co} + \mathrm{Fo}_h'$ for a heating period. One must also consider that Fo_{co}' and Fo_h' have different values for the periods of particle ascent and descent. The separate quantities in (24) are determined as follows: $\nu = i\sqrt{s/\mathrm{Ly}}$, i.e., $s = -\mathrm{Ly}\nu^2$, where s_n are roots of the equation

$$\left(F_{\Gamma}\sqrt{\frac{s}{\mathrm{Ly}}}X\right)'_{X=1} + \mathrm{Bi}_m F_{\Gamma}\sqrt{\frac{s}{\mathrm{Ly}}} = 0, \tag{25}$$

$\mu = i\sqrt{s}$, i.e., $s = -\mu^2$, where in this case s_n are roots of the equation

$$(F_{\Gamma}\sqrt{s}X)'_{X=1} + \mathrm{Bi}\,F_{\Gamma}\sqrt{s} = 0. \tag{26}$$

The values of $\psi_1'(\nu_n)$ and $\psi_1'(\mu_n)$, with the substitution of ν_n or μ_n for s_n, are obtained from the expression

$$\psi_1'(s) = s\left\{\left[\left(F_{\Gamma}\sqrt{\frac{s}{\mathrm{Ly}}}X\right)'_{x=1}\right]'_s + \mathrm{Bi}_m\left(F_{\Gamma}\sqrt{\frac{s}{\mathrm{Ly}}}\right)'_s\right\}$$
$$\times[(F_{\Gamma}\sqrt{s}X)'_{X=1} + \mathrm{Bi}\,F_{\Gamma}\sqrt{s}] + s\left[\left(F_{\Gamma}\sqrt{\frac{s}{\mathrm{Ly}}}X\right)'_{X=1} + \mathrm{Bi}_m F_{\Gamma}\sqrt{\frac{s}{\mathrm{Ly}}}\right]\{[(F_{\Gamma}\sqrt{s}X)'_{X=1}]'_s + \mathrm{Bi}\,(F_{\Gamma}\sqrt{s})'_s\}. \tag{27}$$

We note that when $s_n = -\mathrm{Ly}\nu_n^2$ the second term in (27) equals zero because of (25), and when $s_n = -\mu_n^2$ the first term equals zero because of (26). The value of $\psi_2'(\mu_n)$, with the substitution of μ_n for s_n, is determined from the equation

$$\psi_2'(s) = [(F_{\Gamma}\sqrt{s}X)_{X=1}]'_s + \mathrm{Bi}\,(F_{\Gamma}\sqrt{s})'_s$$

(the index shows the parameter with respect to which the derivative must be taken);

$$\Omega_1 = \mathrm{Bi}\,J_h^{(1)m} + \mathrm{Bi}\,J^{(1)} - \mathrm{Bi}\,\Theta^* J^{(1)}, \tag{28}$$

$$\Omega_2 = \mathrm{Bi}\,J_h^{(2)m} + \mathrm{Bi}\,J^{(2)} - \mathrm{Bi}\,\Theta^* J^{(2)}, \tag{29}$$

where

$$J_{\mathrm{h}}^{(1)m} = T_{\mathrm{h}}\left[[\delta_1 s_m^{(1)} - 1 + \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)(m\,\mathrm{Fo}_{\mathrm{mo}} + \mathrm{Fo}_{\mathrm{up}})\right]\right];$$

$$J_{\mathrm{h}}^{(2)m} = \delta_2 s_{m+1}^{(2)} \exp(-p).$$

The quantity $J^{(i)}$ takes on different values in accordance with (5) and (7) depending on whether heating or cooling of the material occurs and with allowance for the ascent or descent of the particles. In accordance with (7) and Fig. 3, following the preliminary heating there occur the cooling and then the heating of the material, which in an oscillating mode are periodically repeated. For cooling during ascent we have

$$J_{\mathrm{co}}^{(1)\mathrm{up}} = \left\{T_{\mathrm{h}} s_k^{(1)}\left[\delta_1 s_m^{(1)} - 1 + \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)(m\,\mathrm{Fo}_{\mathrm{mo}} + \mathrm{Fo}_{\mathrm{up}})\right]\right]\right.$$
$$\times \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{co}}\right] + T_{\mathrm{co}}\, s_{k+1}^{(1)}\left(\delta_1 s_m^{(1)} - 1 + \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}}\right.\right.\right.$$
$$\left.\left.\left.\left. - \mu_n^2\right)(m\,\mathrm{Fo}_{\mathrm{mo}} + \mathrm{Fo}'_{\mathrm{do}})\right]\right)\right\} \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}}\right]; \tag{30}$$

$$J_{\mathrm{co}}^{(2)\mathrm{up}} = \left[T_{\mathrm{h}} s_k^{(2)} s_{m+1}^{(2)} \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{Fo}_{\mathrm{co}} + T_{\mathrm{co}}\, s_{k+1}^{(2)} s_m^{(2)}\right]\delta_2 \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}} - p\right]. \tag{31}$$

For cooling during descent

$$J_{\mathrm{co}}^{(1)\mathrm{do}} = \left\{T_{\mathrm{h}} s_k^{(1)}\left[\delta_1 s_m^{(1)} - 1 + \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)(m\,\mathrm{Fo}_{\mathrm{mo}} + \mathrm{Fo}_{\mathrm{up}})\right]\right]\right.$$
$$\left.\times \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{co}}\right] + T_{\mathrm{co}}\, s_{k+1}^{(1)} \delta_1 s_m^{(1)}\right\} \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}}\right]; \tag{32}$$

$$J_{\mathrm{co}}^{(2)\mathrm{do}} = \left\{T_{\mathrm{h}} s_k^{(2)} s_{m+1}^{(2)} \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{Fo}_{\mathrm{co}}\right]\delta_2 + T_{\mathrm{co}}\, s_{k+1}^{(2)} (\delta_2 s_m^{(2)} + \delta'_2)\right\} \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}} - p\right]. \tag{33}$$

In Eqs. (30)-(33), $k = 1, 2, 3, \ldots$ in the cofactors to T_{h}; $k = 0, 1, 2, 3\ldots$ in the cofactors to T_{co}. For heating during ascent

$$J_{\mathrm{h}}^{(1)\mathrm{up}} = \left[T_{\mathrm{h}}\left(\delta_1 s_m^{(1)} - 1 + \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)(m\,\mathrm{Fo}_{\mathrm{mo}} + \mathrm{Fo}'_{\mathrm{up}})\right]\right)\right.$$
$$\left.\times \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{co}}\right] + T_{\mathrm{co}}\,(\delta_1 s_m^{(1)} - 1)\right] s_{k+1}^{(1)} \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}}\right], \tag{34}$$

$$J_{\mathrm{h}}^{(2)\mathrm{up}} = \left[T_{\mathrm{h}} \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{F}_{\mathrm{co}}\right] + T_{\mathrm{co}}\right] s_{k+1}^{(2)} \delta_2 s_m^{(2)} \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}} - p\right]. \tag{35}$$

For heating during descent

$$J_{\mathrm{h}}^{(1)\mathrm{do}} = \left\{T_{\mathrm{h}} \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{co}}\right] + T_{\mathrm{co}}\right\} s_{k+1}^{(1)} (\delta_1 s_m^{(1)} - 1) \exp\left[-\left(\frac{p}{\mathrm{Fo}_{\mathrm{up}}} - \mu_n^2\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}}\right], \tag{36}$$

$$J_{\mathrm{h}}^{(2)\mathrm{do}} = \left[T_{\mathrm{h}}\,(\delta_2 s_m^{(2)} + \delta'_2) \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{Fo}_{\mathrm{co}}\right] + T_{\mathrm{co}}\, s_m^{(2)} \delta_2\right] s_{k+1}^{(2)} \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo}_{\mathrm{do}}}\right)\mathrm{Fo}_{\mathrm{h}}^{\mathrm{ma}} - p\right]. \tag{37}$$

In Eqs. (34)-(37) $k = 0, 1, 2, 3\ldots$.

The quantity $J_{\mathrm{mo}}^{(i)}$ ($i = 1, 2$) takes on different values in accordance with (5) depending on in which period, during the ascent of a particle or its descent, the temperature of the material is calculated.

During the ascent of the particle

$$J_{\mathrm{mo}}^{(1)\mathrm{up}} = \delta_1 s_m^{(1)} - 1 + \exp\left[\left(\mu_n^2 - \frac{p}{\mathrm{Fo}_{\mathrm{up}}}\right)(m\,\mathrm{Fo}_{\mathrm{mo}} + \mathrm{Fo}'_{\mathrm{up}})\right];$$
$$J_{\mathrm{mo}}^{(2)\mathrm{up}} = \delta_2 s_m^2 \exp(-p). \tag{38}$$

During the descent of the particles

$$J_{\mathrm{mo}}^{(1)\mathrm{do}} = \delta_1 s_m^{(1)} - 1; \quad J_{\mathrm{mo}}^{(2)\mathrm{do}} = (\delta_2 s_m^{(2)} + \delta_2') \exp(-p). \tag{39}$$

The separate quantities in (30)-(39) are determined as follows:

$$\delta_1 = \exp\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)\mathrm{Fo_{up}} - \exp\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)\mathrm{Fo_{mo}};$$

$$\delta_2 = \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)\mathrm{Fo_{mo}} - \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)\mathrm{Fo_{mo}}$$

$$\delta_2' = \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)(m\,\mathrm{Fo_{mo}} + \mathrm{Fo_{do}} + \mathrm{Fo_{do}'})\right] - \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)(m\,\mathrm{Fo_{mo}} + \mathrm{Fo_{up}})\right];$$

$$s_k^{(1)} = \frac{1 - \exp\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)k\,\mathrm{Fo_c}}{1 - \exp\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)\mathrm{Fo_c}};$$

$$s_k^{(2)} = \frac{1 - \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)k\,\mathrm{Fo_c}}{1 - \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)\mathrm{Fo_c}};$$

$$s_{k+1}^{(1)} = \frac{1 - \exp\left[\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)(k+1)\mathrm{Fo_c}\right]}{1 - \exp\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)\mathrm{Fo_c}};$$

$$s_{k+1}^{(2)} = \frac{1 - \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)(k+1)\mathrm{Fo_c}\right]}{1 - \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)\mathrm{Fo_c}};$$

$$s_m^{(1)} = \frac{1 - \exp\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)m\,\mathrm{Fo_{mo}}}{1 - \exp\left(\mu_n^2 - \frac{p}{\mathrm{Fo_{up}}}\right)\mathrm{Fo_{mo}}};$$

$$s_m^{(2)} = \frac{1 - \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)m\,\mathrm{Fo_{mo}}}{1 - \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)\mathrm{Fo_{mo}}};$$

$$s_{m+1}^{(2)} = \frac{1 - \exp\left[\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)(m+1)\mathrm{Fo_{mo}}\right]}{1 - \exp\left(\mu_n^2 + \frac{p}{\mathrm{Fo_{do}}}\right)\mathrm{Fo_{mo}}}.$$

In Eqs. (30)-(39) only the quantities marked by primes are variables. Thus, by assigning the value of $\mathrm{Fo_{up}'}$ or $\mathrm{Fo_{do}'}$ one can calculate the temperature of the material as a function of its position in the bed at any moment of the period of ascent or descent of a particle during its heating or cooling. The order of calculation by Eq. (24) with allowance for (30)-(39) is as follows: for each m one calculates all n, and for each k one calculates all m.

The roots μ_n and ν_n have been calculated and tabulated [3, 5] for bodies of regular shape (sphere, cylinder, plate). In his next publication the author hopes to analyze the solution for bodies of regular shape with an example of the calculation of the temperature field of a moist particle.

In solving the problem the law of motion of the particles in the fluidized bed or at least the average velocity of the circulating motion of the particles was assumed to be known. In our opinion the opposite formulation is also possible: from the known law of variation of the temperature of a particle one can estimate its movement in the fluidized bed. It seems to us that the measurement of the particle temperature is a simpler task than the study of its motion, it being sufficient to know the particle temperature at any one point. In such a formulation the most laborious operation is the calculation on a computer on the basis of the equations obtained. However, the inverse problem can be considerably simplified for this purpose.

NOTATION

$\mathrm{Bi} = \alpha R/\lambda$	is the Biot number;
$\mathrm{Bi_m} = \alpha_m R/\lambda_m$	is the mass-exchange Biot number;
$\mathrm{Fo} = a\tau/R^2$	is the Fourier number;
$\mathrm{Ki_m} = \gamma_m(\tau)R/a_m\gamma(u_0-u_e)$	is the mass-exchange Kirpichev number;
$\mathrm{Ko} = \rho(u_0-u_e)/c_{ma}(t_h-\vartheta_h)$	is the Kossovich number;
$\mathrm{Ly} = a/a_m$	is the Lykov number;
Pn	is the Posnov number;

$$T=\frac{t(h,\ \tau)-\vartheta_0}{t_h-\vartheta_0};\quad u=\frac{u_0-u}{u_0-u_e};\quad X=\frac{r}{R};\quad p=\frac{\alpha F}{c_pL}H;\quad \Theta=\frac{\vartheta(r,\ h,\ \tau)-\vartheta_0}{t_h-\vartheta_0};\quad Z=\frac{h}{H};\quad \bar{v}_z=\frac{\bar{v}_aR^2}{Ha};$$

a	is the coefficient of thermal diffusivity;
a_m	is the coefficient of moisture diffusion;
c_p, c_{ma}	are the heat capacities of fluidizing agent and of particle material, respectively;
d	is the particle diameter;
F	is the particle surface per unit bed height;
Fo_h^{ma}	is the dimensionless time of preliminary heating of material;
h, H	are the current and total heights of fluidized bed;
L	is the flow rate of fluidizing agent per unit time;
R	is the characteristic size of elements of bed (particles);
t	is the temperature of heat-transfer agent;
$0 \le r \le R$; u, u_0, u_e	are the current, initial, and equilibrium moisture contents of material;
$\bar{v}_a$	is the average velocity of particle motion;
α	is the coefficient of heat exchange between fluidizing agent and particles;
Γ	is the particle shape factor;
γ	is the specific weight of particle material;
γ_m	is the mass flux;
ϑ, ϑ_0	are the current and initial particle temperatures, respectively;
λ, λ_m	are the coefficients of thermal conductivity and mass conductivity, respectively;
$\mu_n^* = \mu_n/\sqrt{\mathrm{Ly}}$; $\nu_n^* = \nu_n/\sqrt{\mathrm{Ly}}$; ρ	is the phase-transition heat;
τ	is the time.

Indices

h	is the heating;
co	is the cooling;
up	is the up;
do	is the down;
c	is the cycle;
mo	is the motion.

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THEORY OF TWO-PHASE TRANSPIRATION COOLING SYSTEMS. II

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A statistical mode of bubbling in porous solids is formulated. The theory developed is used for closing the system of transport equations for a two-phase transpiration cooling system.

We shall consider here the problem of hydrodynamics and heat exchange in two-phase transpiration cooling systems. In our previous paper [1], we considered the case of a porous solid consisting of capillaries with equal or variable cross sections. However, although this model is of practical interest [2], the more often encountered porous materials with a highly complicated void structure remain outside the scope of applicability of the developed theory. We shall consider here the model of liquid bubbling in porous systems of the fluidized-solid and use it as a basis for writing the macroscopic continuum equations which describe the hydrodynamics and heat exchange in two-phase transpiration cooling systems.

Statistical Model of Bubbling in a Porous Solid

The bubbling of a liquid filtering through a porous material occurs in the following manner. A vapor bubble which has formed in a certain pore grows in volume, fills the entire pore, and then passes into the neighboring pores along the links connecting this pore with the others. As a result, a vapor-filled cavity forms inside

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*Deceased.

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